Polydispersity effects in the dynamics and stability of bubbling flows

E. Salinas-Rodríguez^{1,a,b}, R.F. Rodríguez^{2,c}, J.M. Zamora^{1,b}, and A. Soria^{1,b}

¹ Departamento I. P. H., Universidad Autónoma Metropolitana, Iztapalapa. Apdo. Postal 55–534, 09340 México, D. F., México

² Instituto de Física. Universidad Nacional Autónoma de México, Apdo. Postal 20-364, 01000 México, D. F., México

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Abstract. The occurrence of swarms of small bubbles in a variety of industrial systems enhances their performance. However, the effects that size polydispersity may produce on the stability of kinematic waves, the gain factor, mean bubble velocity, kinematic and dynamic wave velocities is, to our knowledge, not yet well established. We generalize the model by Biesheuvel and Gorissen [10] to incorporate size polydispersity of bubbles. We find that it enhances the stability of a bubble column by a factor of about 23% as a function of frequency. In this way our model predicts effects that might be verified experimentally; however, this remain to be assessed. Our results reinforce the point of view advocated in this work in the sense that a description of a bubble column based on the concept of randomness of a bubble cloud and average properties of the fluid motion, may be a useful approach that has not been exploited in engineering systems.

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1 Introduction

The theoretical description of multiphase flows is essentially based on analyzing the response of a cloud of dispersed particles of different size ranges in a fluid. These particles constitute dynamic phases and hence a multiphase flow. A widely used multiphase system is a bubble column which is a reactor where a discontinuous gas phase in the form of bubbles, moves relative to a continuous phase. Bubble columns have a wide range of applications in chemical industries, biotechnology or in nuclear reactors [1-5]. The transient behavior is important at the start-up of these systems and its analysis is essential in order to characterize the dynamic performance of the columns. Among the phenomena that occur in these systems void wave propagation mechanisms are of great importance since many transient and steady states are controlled by the propagation of these waves and, in this sense among others, the dynamic characterization of multiphase flows is essential for the prevention of instabilities.

The (in)stability of bubbly flows which are characterized by almost uniformly sized bubbles, is usually de-

scribed in terms of the propagation properties of void fraction and pressure disturbances caused by natural or imposed fluctuations of the rate of air supply [6–8]. Bubble size, rise velocity, size distribution and liquid and bubble velocity profile have a direct bearing on the performance of bubble columns. However, most of the time the dispersion devices deliver a dispersed phase with a given size distribution. The importance of the size distributions is only scarcely evaluated, most of the time by direct empirical trials and its influence on the global behavior has still to be studied. Actually, to our knowledge and from the theoretical point of view, it has not been yet well established whether the stability of the motion of a swarm of bubbles is different for monodispersed or polydispersed bubble flows. The main objective of this work is to investigate the effects that size polydispersity might produce on the stability of a bubble column. We shall introduce the effect of polydispersity through the drag force in the hydrodynamic equations, using a method based on statistical concepts and on a point-force approximation [9]. As we shall see below, the corrections on the drag force factor, $C_D^P(a)$, due to polydispersity depend only on the first three moments of a given particle size distribution and they also have an effect on several properties of kinematic waves. In particular, we found that size polydispersity enhances the stability of void waves by a factor which varies

^a e-mail: sabe@xanum.uam.mx

^b Fellow of SNI, Mexico.

^c Also at FENOMEC. Fellow of SNI, Mexico.

between 4.5-23% as a function of frequency and for a particular type of bubble column. In this way our model predicts effects that might be verified experimentally but this, however, remains to be assessed.

To this end the paper is organized as follows. In Section 2 we briefly review a hydrodynamic model for bubbly fluids introduced by Biesheuvel and Gorissen [10]. Next, in Section 3 we consider a dispersion of spherical air bubbles of different radii in water and we calculate the effect of size polydispersity on the gain factor, mean bubble velocity, kinematic wave velocities as a function of void fraction, for different wave frequencies.

2 Equations of motion of a bubble dispersion

In this section we summarize the main ideas and steps behind the hydrodynamic model for bubbly fluids introduced in reference [10]. The equations of motion for a swarm of bubbles in a bubble column have been derived in the literature by using standard methods of kinetic theory to average over an ensamble or realizations of the flow [11, 12]. In reference [10] a dispersion of equally sized air bubbles in a water column where the bubbles are small enough to remain spherical through the whole system, is considered. They assumed that the air can be taken as an incompressible fluid where no mass transfer is allowed between the bubbles and the water, which is assumed to be an incompressible Newtonian liquid. The conservation equation for the mean number density of the gas bubbles, n, and the conservation equation for the mean bubble momentum, $\rho_G \mathbf{v}$ (Kelvin impulse), were obtained for this system [13],

$$\frac{\partial n}{\partial t} + \nabla_{\mathbf{x}} \cdot (n\mathbf{v}) = 0 \tag{1}$$

$$\frac{\partial}{\partial t} \left[n \left(\frac{4}{3} \pi a^3 \rho_G \mathbf{v} + \mathbf{I}_L \right) \right] + \nabla_{\mathbf{x}} \cdot \left[n \left(\frac{4}{3} \pi a^3 \rho_G \mathbf{v} + \mathbf{I}_L \right) \mathbf{v} \right] - \nabla_{\mathbf{x}} \cdot \left(\mathbb{T}_G + \mathbb{T}_L \right) = n \mathbf{F}_D + n \frac{4}{3} \pi a^3 (\rho_L - \rho_G) \mathbf{g}. \quad (2)$$

 \mathbf{I}_L is the fluid impulse, $\mathbb{T}_L(\mathbf{x},t)$ and $\mathbb{T}_G(\mathbf{x},t)$ are the fluid stresses; \mathbf{F}_D is the drag force exerted by the fluid on the bubble and \mathbf{g} stands for the gravity field; ρ_L , ρ_G denote, respectively, the mass densities of water and air. μ_L stands for the liquid viscosity. In order to describe the flow parameters of the bubble swarm, equations (1) and (2) should be expressed in terms of the volume fraction of bubbles (or void fraction) ε and their velocity field \mathbf{v} . Following reference [10] we assume that the uniform flow of bubbles is along the axial direction of the column with a mean axial rise velocity $v_0(\varepsilon)$, Therefore, $\varepsilon(z,t) \equiv \frac{4}{3}\pi a^3 n(z,t)$.

The effect of hydrodynamic interactions between the bubbles on the mean frictional force may be represented by introducing a function $f_0(\varepsilon)$ into $v_0(\varepsilon)$ in the form $v_0(\varepsilon) = f_0^{-1}(\varepsilon)v_{\infty}$, [10]. The magnitude of the terminal velocity,

 v_{∞} , of a single bubble of radius a in a stagnant liquid is given by [16] $v_{\infty} \equiv C_D^{-1}(\rho_L - \rho_G)g$, where $C_D \equiv 9\mu_L/a^2$ is the drag force factor and experiments suggest that [14],

$$f_0(\varepsilon) = (1 - \varepsilon)^{-2}.$$
(3)

The mean fluid impulse is modelled by

$$nI_L = n\left(\frac{2}{3}\pi a^3\rho_L\right)m_0(\varepsilon)v_0(\varepsilon),\tag{4}$$

where $m_0(\varepsilon)$ takes into account the effect of the hydrodynamic interactions. According to reference [15] an expression for $m_0(\varepsilon)$ that renders reliable results up to large values of ε is $m_0(\varepsilon) = (1 + 2\varepsilon)/(1 - \varepsilon)$.

Since in a nonuniform bubbly flow the stress $\mathbb{T} = \mathbb{T}_G + \mathbb{T}_L$ play the role of an effective pressure, they also assume that the *kinetic* contribution, $p_e(\varepsilon)$, is proportional to the effective density of the bubbles, $\varepsilon^{-1}\rho_{ef}(\epsilon) \equiv \rho_G + \frac{1}{2}\rho_L m_0(\varepsilon)$, and to the mean square of their velocity fluctuations $\overline{\Delta v^2} \equiv H(\varepsilon)v_0^2(\varepsilon) = \frac{\varepsilon}{\varepsilon_{cp}}\left(1 - \frac{\varepsilon}{\varepsilon_{cp}}\right)v_0^2(\varepsilon)$, [16]. Here ε_{cp} stands for the limit of closest packaging of a set of spheres and is close to the value 0.62. Thus, $p_e(\varepsilon) = \rho_{ef}\overline{\Delta v^2}$. Furthermore, if the non-uniformity is the main cause of an additional transfer of bubble momentum and fluid impulse associated with stress, Biesheuvel and Gorissen [10] postulate that such a contribution to the stress should be given by the force $\mu_e(\varepsilon)\frac{\partial}{\partial z}v$. Therefore, taking into account both contributions to the stress, $\mathbb{T} = -p_e(\varepsilon) + \mu_e(\varepsilon)\frac{\partial v}{\partial z}$, where v is the one dimensional nonuniform flow velocity and $\mu_e(\varepsilon) = a\rho_{ef}(\varepsilon)v_0(\varepsilon)H^{1/2}(\varepsilon)$ is an effective viscosity.

On the other hand, the mean frictional force is enhanced by an effective diffusive flux of bubbles due to their fluctuating motion. This effect is similar to an steady drag force acting upon each one of the bubbles and proportional to the mean number density gradient. Therefore, using (2) this force is represented by $nF_D = C_D \varepsilon f_0(\varepsilon) [v + \frac{\mu_e(\varepsilon)}{\varepsilon} \frac{\partial \varepsilon}{\partial z}]$. Substitution of the above expressions into equations (1) and (2) leads to the following closed set of one-dimensional equations of motion for the bubbly flow in a zero volume flux reference frame,

$$\frac{\partial\varepsilon}{\partial t} + \frac{\partial}{\partial z}(\varepsilon v) = 0, \tag{5}$$

$$\frac{\partial}{\partial t} \left[\rho_{ef}(\varepsilon) v \right] + \frac{\partial}{\partial z} \left[\rho_{ef}(\varepsilon) v^2 \right] - \frac{\partial}{\partial z} \mathbb{T} \\
= -C_D \varepsilon f_0 \left(v + \frac{\mu_e(\varepsilon)}{\varepsilon \rho_{ef}} \frac{\partial \varepsilon}{\partial z} \right) - \varepsilon \left(\rho_G - \rho_L \right) g. \quad (6)$$

These equations may be rewritten in a laboratory reference frame by considering the mean axial velocity of the dispersion, U, defined by $U(t) \equiv \varepsilon U_G + (1 - \varepsilon) U_L$. Here U_G and U_L are the mean bubble and fluid axial velocity in the laboratory reference frame. Note that due to the incompressibility of both, liquid and gas, U is only a function of time. Therefore $v \equiv U_G - U$ and a Galileo transformation of equations (5) and (6) gives

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial z} \varepsilon U_G = 0, \tag{7}$$

$$\frac{\partial}{\partial t} \left[\varepsilon \left(\rho_G U_G + \frac{1}{2} \rho_L m_0 \left(U_G - U \right) \right) \right] \\
+ \frac{\partial}{\partial z} \left[\varepsilon \left(\rho_G U_G + \frac{1}{2} \rho_L m_0 \left(U_G - U \right) \right) U_G \right] \\
- \frac{\partial}{\partial z} \left(-p_e + \mu_e \frac{\partial U_G}{\partial z} \right) - \varepsilon \rho_G \frac{\partial U}{\partial t} \\
= -C_D \varepsilon f_0 \left(\left(U_G - U \right) + \frac{\mu_e(\varepsilon)}{\varepsilon \rho_{ef}} \frac{\partial \varepsilon}{\partial z} \right) - \varepsilon \left(\rho_G - \rho_L \right) g,$$
(8)

together with the incompressibility condition

$$\frac{\partial U}{\partial z} = 0. \tag{9}$$

Consider a quiescent equilibrium state of the dispersion described by $\varepsilon = \varepsilon_0$. The deviations from this state will be denoted by $\delta \varepsilon(z,t)$ and $\delta v(z,t)$. Linearization of equations (7)–(9) around the reference state yields the wavehierarchy equation

$$\tau_{\varepsilon} \left[\left(\frac{\partial}{\partial t} + c^{+} \frac{\partial}{\partial z} \right) \left(\frac{\partial}{\partial t} + c^{-} \frac{\partial}{\partial z} \right) \\ \times \delta \varepsilon - \nu_{\varepsilon} \left(\frac{\partial}{\partial t} + U_{G0} \frac{\partial}{\partial z} \right) \frac{\partial^{2} \delta \varepsilon}{\partial z^{2}} \right] \\ = - \left[\left(\frac{\partial}{\partial t} + c_{0} \frac{\partial}{\partial z} \right) \delta \varepsilon - \nu_{\varepsilon} \frac{\partial^{2} \delta \varepsilon}{\partial z^{2}} \right]$$
(10)

with lower and higher-order wave velocities given by $c_0 \equiv U_{G0} + \varepsilon_0 v'_0$ and

$$c^{\pm} \equiv U_{G0} - \frac{\frac{1}{4}\varepsilon_{0}\rho_{L}v_{0}m_{0}'}{\rho_{G} + \frac{1}{2}\rho_{L}m_{0}} \\ \pm \left[\left(\frac{\frac{1}{4}\varepsilon_{0}\rho_{L}v_{0}m_{0}'}{\rho_{G} + \frac{1}{2}\rho_{L}m_{0}} \right)^{2} + \frac{p_{e}'}{\rho_{G} + \frac{1}{2}\rho_{L}m_{0}} \right]^{1/2}.$$
 (11)

Here $\nu_e(\varepsilon) = v_0(\varepsilon)H^{1/2}(\varepsilon)$ and $\tau_e(\varepsilon) = (C_D f_0)^{-1} \left[\rho_G + \frac{1}{2}\rho_L m_0(\varepsilon)\right]$. The primes (\prime) denote derivatives with respect to ε and evaluated at the unperturbed state $\varepsilon = \varepsilon_0$.

For relatively low radial frequencies the wave propagation is described by a linearized Burgers/Kortewegde Vries equation

$$\left(\frac{\partial}{\partial t} + c_0 \frac{\partial}{\partial z}\right) \delta \varepsilon \approx \left[\tau_{\varepsilon} (c^+ - c_0)(c_0 - c^-) + \nu_{\varepsilon}\right] \frac{\partial^2 \delta \varepsilon}{\partial z^2} + \tau_{\varepsilon} \nu_{\varepsilon} (U_{G0} - c_0) \frac{\partial^3 \delta \varepsilon}{\partial z^3}, \quad (12)$$

with a solution $\varepsilon \propto \exp(\gamma z - i\omega t)$ where ω is the frequency of the void wave and

$$\gamma \approx \frac{i\omega}{c_0} \left[1 - \frac{\nu_{\varepsilon} \tau_{\varepsilon} \omega^2 \left(U_G - c_0 \right)}{c_0^3} \right] - \frac{\nu_{\varepsilon} \omega^2 \left(U_G - c_0 \right)}{c_0^3} \left[\tau_{\varepsilon} (c^+ - c_0) (c_0 - c^-) + \delta_{\varepsilon} \right].$$
(13)

In terms of these quantities the so called gain factor, $G_f \equiv \exp \left[\operatorname{Re}(\gamma)\omega^2 \Delta z\right]$, where $\operatorname{Re}(\gamma)$ denotes the real part and Δz the distance between two impedance probes in the experiments to measure G_f [18].

3 Polydispersed dispersion

The method developed by Tam [9] uses the concept of randomness of the bubble cloud and derives equations describing the average properties of the fluid motion. These averages are taken over a statistical ensemble of particle configurations. A slow viscous flow past a large collection of spheres of a given size distribution is considered to derive a particle drag formula free from empirical assumptions. The result essentially replaces the disturbance produced by a sphere in low Reynolds number flow, by that of a point force located at the centre of the sphere. The correction drag force factor is given by

$$C_D^P = \lambda C_D \equiv \left[1 + \alpha \overline{a} + \frac{1}{3} (\alpha \overline{a})^2\right] C_D, \qquad (14)$$

where

$$\alpha = \frac{6\pi M_2 + \left[\left(6\pi M_2 \right)^2 + 12\pi M_1 (1 - 3c) \right]^{1/2}}{(1 - 3c)}.$$
 (15)

 $M_n = \int n(a)a^n da$ are the moments of the size distribution n(a) and $c \equiv \frac{4}{3}\pi M_3$.

Since the terminal velocity of a bubble depends on C_D , it is reasonable to assume that in the polydispersed case $v_0(\varepsilon)$ should be replaced by $v_0^P \equiv \lambda^{-1}v_0$. Substitution of this assumption into equations (7)–(9), carrying out the linearization procedure described in the last section and using the explicit expressions of $\beta \equiv$ $\{U_{G0}, c_0, c^{\pm}\}$, one can show that these quantities scale as $\beta^p \equiv \{\lambda^{-1}U_{G0}, \lambda^{-1}c_0, \lambda^{-1}c^{\pm}\}$. If these polydispersed quantities are substituted into equation (13), one obtains an expression for the polydispersed gain factor $G_f^p \equiv$ $\exp\left[\operatorname{Re}(\gamma^p)\omega^2\Delta z\right]$.

4 Results

To compare the monodispersed (MD) and polydispersed (PD) results on the gain factor (G_f, G_f^p) , mean bubble velocity (U_{G0}) , kinematic wave velocity (c_0) and dynamic wave velocities (c^{\pm}) as functions of the void fraction ε , we used the following material parameter values for



Fig. 1. The monodispersed G_f (*MD*) and polydispersed G_f^p (*PD*) gain factors vs. ε for waves with frequencies of 2, 2.5, 3 Hz. The liquid is stagnant and the material parameter values are those given in Section 4.

an air-water bubble column, namely, $\Delta z = 20 \text{ cm}$, $V_T = 1280 \text{ cm}^3$, $\rho_G = 1.2046 \times 10^{-3} \text{ gr/cm}^3$, $\rho_L = 0.998 \text{ gr/cm}^3$, $\mu_L = 1.002 \times 10^{-2}$ poise, $\varepsilon_{cp} = 0.62$. In Figure 1 we plot both, the gain factor G_f for the monodispersed (*MD*) and G_f^p for the polydispersed (*PD*) cases vs. ε for different frequencies $\omega = 2, 2.5, 3$ Hz. The size polydispersity is described by a log-normal distribution n(a) with average $\overline{a} = 0.04$ cm and dispersion $\sigma = 0.5$.

Note that for values $0.185 \leq \varepsilon \leq 0.301$ the attenuation rate drops significantly. For instance, the percentage difference defined by $\Gamma \equiv |G_f - G_f^p|/G_f$ for a frequency of 2 Hz, ranges from 0.1-4.98%, whereas for a frequency of 3 Hz it varies in the interval 0.1-22.78%. This means that stability is larger in about 23% for the latter case, a change that is significant in bubble reactors [17].

The quantities $\beta \equiv \{U_{G0}, c_0, c^{\pm}\}$ and $\beta^p \equiv \{\lambda^{-1}U_{G0}, \lambda^{-1}c_0, \lambda^{-1}c^{\pm}\}$ are plotted as functions of ε in Figure 2. Note that in agreement with the Whitham stability criterion [19], the curve for c_0 is always between that for $c^+(PD)$ and $c^-(PD)$ in the interval $0 < \varepsilon < 0.299$ and is therefore stable. When $c_0 < c^-$ in the range $0.299 < \varepsilon < 0.4$, the flow is unstable. In contrast, for the monodispersed case it is stable between $0 < \varepsilon < 0.302$, whereas it becomes unstable for $\varepsilon > 0.302$.

5 Discussion

Summarizing, in this work we have analyzed the effects of size polydispersity in several features of the void fraction waves and their stability properties. We found that the presence of a size distribution reinforces the stability of the waves, as shown in Figures 1 and 2. Furthermore, the percentage difference $\Gamma \equiv |\beta - \beta^p| / \beta$ turns out to be $\Gamma = 4.9\%$.

It is convenient to emphasize once again, that the hydrodynamic model used in this work [10] is idealized in many aspects. For instance, compressibility and hydrodynamic interactions between bubbles and with the bound-



Fig. 2. $\beta \equiv \{U_{G0}, c_0, c^{\pm}\}$ and $\beta^p \equiv \{\lambda^{-1}U_{G0}, \lambda^{-1}c_0, \lambda^{-1}c^{\pm}\}$ as functions of the void fraction ε for the same parameter values as in Figure 1.

aries, have not been taken fully into account. The liquid phase is incompressible implying that its density is not a function of space and time. However, the bubble dispersion is compressible because the local hold-up is not a constant. The compressibility could be accounted for by the spatial variation of the fractional gas hold-up. Although the effect of hydrodynamic interactions between bubbles on the mean frictional force have been partially accounted for through the function $f_0(\varepsilon)$ in equation (3), other hydrodynamic interactions have been neglected. A possible way of including these interactions would be through the consideration of the disturbed forces like Basset-Boussinesq forces and by incorporating the spatial inhomogeneities of the fluid velocity field by Faxen's terms. However, given the complexity of these effects and of the system itself, the simple dimensional model proposed by Biesheuvel and Gorissen seems to be a good first step in modeling the complex behavior of a bubble column. It also illustrates how some of the methodology and concepts of kinetic theory and statistical mechanics may be used to deal with complex phenomena in engineering systems.

We should also mention that in this work we have assumed an initial polydisperse size distribution and the coalescence and fragmentation of bubbles has not been considered [20,17]. However, the validity of this assumption remains to be assessed. Nevertheless the mean field approach considered here by including the influence of the distribution through the drag effects, is an attempt to set a first framework to incorporate bubble size effects in future studies. Our results reinforce this point of view in the sense that a description of a bubble column based on the concept of randomness of a bubble cloud and average properties of the fluid motion, may be a useful approach that has not been exploited in engineering systems.

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